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Experimental verification of a strange transformation for point rotation coordinate frames

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Abstract
We consider the general form of linear transformation for point rotation coordinate frames. The frames have the rotation axis at every point. In the transformation, the frequency of one frame relative to another is not equivalent to the reverse frequency. Using symmetry of the direct and reverse transformation as well as symmetry of the frame coordinates, we show that three different types of transformation are possible. The first type is a generalization of the Lorentz transformation. This case cannot be checked in optical measurements. In contrast to this, the second unusual type allows us to observe consequences of the transformation in an optical experiment even though the characteristic constant inherent in the transformation is about 'nuclear time' of the order of 10^{-23} s. We describe the schematic of the experiment. The third type is a supplement to the second type. This type is not applicable to the electromagnetic field and cannot be verified optically.

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1. Introduction

The concept of a 'point rotation frame' arises in crystallooptics. A distinctive feature of the frame, in contrast to a Cartesian one, is the existence of the rotation axis at every point. In such a frame, axes are constructed on field amplitudes and only the axis direction is essential. Similar (non-rotating) frames have been used in quantum field theory for a long time. An example of the frame is the rotating optical indicatrix (index ellipsoid). One more difference in a comparison with the Cartesian frame is the absence of centrifugal forces in the point rotation frame. The frame coordinates are an angle (phase) and time; the frequency of rotation is a parameter. In electro-optical crystals the rotation is stimulated by an applied rotating electric field [1, 2]. In crystals with the linear (Pockels) effect, the frequency equals a half (and may be reduced to a quarter [3]) of that of the electric field. In the Kerr crystals, this frequency is doubled. The sense of rotation of the plane circularly polarized light wave moving through the electro-optical crystal with the rotating optical indicatrix is reversed, and the optical frequency is shifted if the amplitude of the applied electric field equals the half-wave value. The device for the shift by means of electro-optical crystals is the single-sideband modulator [4–6]. Note that optically the rotating phase plate is equivalent to the modulator, but physically it is different as the plate has only one axis of rotation whereas the resting crystal has an axis at every point.

It is convenient to use for the description of circularly polarized plane light wave in the single-sideband modulator the transition to a frame with the resting optical indicatrix [5]. Such a transition results in change of the optical frequency. The change equals the frequency of rotation of the optical indicatrix. After the polarization reversal and returning back to the initial frame, the frequency deviation is doubled. Note that in the frame with the resting indicatrix, the modulating electric field is also at rest in spite of the fact that both the indicatrix and field rotate at different frequencies relative to the initial frame. This is one further unusual and strange property of the point rotation frames.

The transition to the rotating frame is always connected with the question of what is the frequency superposition law, is it linear or not? The nonlinear law always corresponds to an extra frequency shift. It should be emphasized that the consideration in the framework of Maxwell's equations cannot give such an extra shift. The situation is analogous to a comparison between results obtained for rectilinear motion with the help of the Lorentz transformation and Newtonian mechanics.

In [7] the question was considered on the assumption that the combined frequency may be presented in terms of power series of two other frequencies. It was also assumed that the frequency of one frame relative to another is equal to the reverse frequency. The only difference is the sign. The negative sign corresponds to the rotation in the opposite direction. Under these conditions, the extra shift in the first approximation is proportional to the product of the optical frequency squared and the modulation frequency. The characteristic constant in the extra shift has dimension of time. In [7] an optical experiment was also proposed for measurement of the term, and it was shown that a lower limit for measurements of the characteristic constant with such a form of the shift is about 10^{-17} and 10^{-20} s for modulating frequencies $\sim$1 GHz and optical frequencies, respectively.

Shortly, it was shown that an analogy exists between the light propagation in a medium with the rotating optical indicatrix and the motion of a particle in the rotating magnetic field and that both the phenomena can be described in the framework of the Pauli equation [8]. In other words, a plane circularly polarized light wave propagating along the optical axis of a three-fold electro-optical crystal under the action of an applied electric field possesses properties of a two-component spinor. It means that measurements of the optical frequency shift in the single-sideband modulator are similar to measurements of the magnetic moment in the magnetic resonance, and the anomalous magnetic moment may be associated with the nonlinear frequency shift. It was understood that the probable value of the characteristic constant is at most of the order of nuclear time $\sim 10^{-23}$ s [8]. Such a small value excludes the possibility of observing in optical experiments the term calculated in [7].

Meanwhile, the immediate way to determine the frequency superposition law is the transformation for the point rotation frames. In spite of the fact that the concept of the optical indicatrix arises from Maxwell's equations, the equations do not contain any information about the transformation. The transformation must be postulated.

In this paper we consider general linear transformations for the point rotation frames. We use the symmetry of the frame coordinates and assume that the reverse frequency is a function of the direct frequency with the same function and vice versa. We show that three different types of transformation exist. The first type is a generalization of the Lorentz transformation.

This type, in an experimental sense, corresponds to the case of [7]. Both the second and third previously unknown types are essentially different. We consider unusual physical properties of the transformations. Allowable frequencies of the third type are bounded below; therefore, this type is not applicable to the electromagnetic field. In contrast to this, the second type gives us a chance to measure the extra shift. We describe a schematic of an experiment for such a measurement. The experiment keeps the main features of that in [7]. A distinctive feature of the schematic is the use of resonance which can be realized in the given case.

As noted above, the point rotation frames are similar to frames of the quantum field theory. Therefore, the properties must be applicable to quantum fields. This issue however is beyond the scope of this paper.

2. General linear transformation

The general form of the linear transformation for the transition from one frame to another can be written as follows:

$$\tilde{\varphi} = q(\varphi - \nu t), \qquad \tilde{t} = \frac{\tilde{q}q - 1}{\tilde{q}\tilde{\nu}}\varphi - q\frac{\nu}{\tilde{\nu}}t, \tag{1}$$

where φ and t are angle (phase) and time, respectively, and the tilde corresponds to the reverse transformation

$$\varphi = \tilde{q}(\tilde{\varphi} - \tilde{\nu}\tilde{t}), \qquad t = \frac{\tilde{q}q - 1}{q\nu}\tilde{\varphi} - \tilde{q}\frac{\tilde{\nu}}{\nu}\tilde{t}, \tag{2}$$

where ν is the frequency of the second frame relative to the first one. It is obvious that equation (1) turns into equation (2) if variables with the tilde change to variables without the tilde and vice versa.

First of all we exclude from consideration the Galilean transformation, i.e. the case $q \equiv 1$. This case with its infinite frequencies seems unbelievable from the viewpoint of contemporary physics.

Normalizing,

$$\varphi \to \varphi\sqrt{|\tilde{\nu}/\nu|}, \quad t \to t, \quad q \to q|\nu/\tilde{\nu}|, \quad \nu \to \sqrt{|\tilde{\nu}\nu|}$$

or

$$\varphi \to \varphi\sqrt{|\tilde{\nu}|}, \quad t \to t\sqrt{|\nu|}, \quad q \to q\sqrt{|\nu/\tilde{\nu}|}, \quad \nu \to \sqrt{|\tilde{\nu}\nu|},$$

etc, we would arrive at the case $\nu = -\tilde{\nu}$. However, we cannot carry out arbitrary normalization since usually t and φ are already determined and connected with the space Cartesian coordinates. Generally, the point rotation frames are not compatible with the Cartesian frame except when the frames are at rest. In this case if the rotation axis coincides with some Cartesian axis, we may associate φ and t with the Cartesian cylindric angle and time. The above normalization would result, in particular, in a change of the speed of light. Therefore, we must consider here the general case assuming that $\tilde{\nu}$ is a function of ν. It means that rotations to the right and left are not equivalent in the approach.

For the point rotation frame, we do not have a general principle like the relativity principle for the Cartesian frames; however we use the principle of symmetry instead. It means, in particular, that if $\tilde{\nu}(\nu) = f(\nu)$ then $\nu(\tilde{\nu}) = f(\tilde{\nu})$.

The functions $\tilde{\nu}(\nu)$ and $q(\nu)$ remain indeterminate except for the condition at small ν, namely, $\tilde{\nu} \to -\nu, q \to 1$ if $\nu \to 0$. If the characteristic constant τ is of the order of 10^{-23} s, then the normalized frequency $\tau\nu$ even in a microwave range is about 10^{-12}. Therefore, we assume that the function $\tilde{\nu}(\nu)$ may be expanded in a power series in ν:

$$\tilde{\nu}(\nu) = -\nu + a_2\nu^2 + a_3\nu^3 + a_4\nu^4 + \cdots. \tag{3}$$

This expansion is compatible with the reverse expansion under certain conditions for the coefficients a_n, namely, the expansion cannot contain only odd powers of ν and up to the term ν^{2n} has only n independent coefficients a_n. It is obvious that any expansion (3) together with the reverse expansion can be written in a symmetric form with the help of a finite or infinite series

$$\tilde{\nu} + \nu = \sum_{n=1} b_n (\tilde{\nu}\nu)^n \equiv F(\tilde{\nu}\nu), \tag{4}$$

where F is a function. The results below will be also valid for arbitrary $F(\tilde{\nu}\nu)$ with the only condition $F(0) = 0$.

The main problem in the given approach is the nonlinear frequency shift. However since it is defined by the product $\tilde{q}q$, we do not need to know the explicit form of the function $q(\nu)$. For finding the form of $\tilde{q}q$, we use the symmetry about φ and t. Transformation (1) can be written as

$$\tilde{t} = Q(t - \Lambda\varphi), \qquad \tilde{\varphi} = \frac{\tilde{Q}Q - 1}{\tilde{Q}\tilde{\Lambda}} t - Q\frac{\Lambda}{\tilde{\Lambda}}\varphi, \tag{5}$$

where the role of (φ, t, q, ν) is played by (t, φ, Q, Λ) and

$$Q = -q\frac{\nu}{\tilde{\nu}}, \qquad \Lambda = \left(1 - \frac{1}{q\tilde{q}}\right)\frac{1}{\nu}. \tag{6}$$

It is natural to assume that the equality in equation (4) would still be valid if Λ/σ, $\tilde{\Lambda}/\sigma$ is substituted for ν, $\tilde{\nu}$. Here, σ is a dimensional constant. Making use of the substitution and excluding $(\tilde{\nu} + \nu)$, we obtain the equation for $\tilde{q}q$

$$F\left(\frac{\Theta^2}{\tilde{\nu}\nu}\right) - \frac{\Theta}{\tilde{\nu}\nu}F(\tilde{\nu}\nu) = 0, \tag{7}$$

where $\Theta = (1 - 1/q\tilde{q})/\sigma$. Equation (7) is retained under the change $\nu \to \Theta/\nu$; therefore, any solution of the equation is also kept under this change.

Three types of solutions of equation (7) exist. The first type is the exact solution $\Theta = \tilde{\nu}\nu$ or

$$1 - \frac{1}{q\tilde{q}} = \sigma\tilde{\nu}\nu, \tag{8}$$

where $\sigma = \pm\tau^2$. The above invariance under the change $\nu \to \Theta/\nu$ is particularly easy to see for solution (8).

Transformation (1) for the solution of the first type is a generalization of the Lorentz transformation. From the viewpoint of experimental checking, this case is equivalent to the results of [7].

The second type of solutions may be presented as a series

$$1 - \frac{1}{\tilde{q}q} \equiv \sigma\Theta = \sigma c|\tilde{\nu}\nu|^{\frac{1}{2}} + \sigma\sum_{n=2} c_n|\tilde{\nu}\nu|^{\frac{1}{2}n}, \tag{9}$$

where $c = \pm\sqrt{|r|}$, c_n are constants, r is a negative root of equation $F(r) = 0$. At the zeros of F an exact equality $\tilde{\nu} = -\nu$ holds, i.e. values $\nu = \pm\sqrt{|r|}$ are peculiar points. Solution (9) may be written as a function of ν or $\tilde{\nu}$. If $\Theta(\nu)$ is a solution, then the reverse function $\tilde{\Theta}(\nu) \equiv \Theta(\tilde{\nu})$ is also a solution.

Numerical calculations show that a third type of solution as a supplement to the second type is possible. Both the types, at least for the function F with a finite number of terms in expansion (4), have upper boundaries for allowable values of ν. Moreover, the third type has also lower boundaries of ν. Therefore, this type is not applicable to the electromagnetic field.

Assuming the general character of transformation (1), we may associate different solutions with different fields. The number of solutions depend on the form of F. The lower frequency boundaries may then be associated with field masses. In contrast to the Lorentz transformation with only one upper boundary value of the velocity, in the given approach, different fields possess different upper boundary values of the frequency.

Solutions of the first, second and third types are possible if the number of terms in expansion (4) is not less than 1, 2 and 3 respectively.

The function Θ plays an essential role in the nonlinear frequency superposition law (see section 3). The first term in the right part of equation (9) determines the law in the first approximation. The characteristic constant in the given case is

$$\tau = \pm\sigma\sqrt{|r|}. \tag{10}$$

For simplicity, we use the same letter for the characteristic constant in both the types of solutions.

The second type of solution adds to the list one further strange property of the point rotation frames. The normalization

$$\nu^* = \sqrt{-\tilde{\nu}\nu}, \qquad \varphi^* = \varphi\frac{\nu^*}{\nu}, \qquad t^* = t, \qquad q^* = q\frac{\nu}{\tilde{\nu}} \tag{11}$$

imparts the Lorentz shape to equation (1)

$$\tilde{\varphi}^* = q^*(\varphi^* - \nu^* t), \qquad \tilde{t}^* = q^*(-\tau\nu^*\varphi^* + t). \tag{12}$$

If $\nu^* \to 0$, then $\varphi^* \to \infty$. On the other hand, the non-normalized transformation at $\nu \to 0$ tends to the Galilean form

$$\tilde{\varphi} = \varphi, \qquad \tilde{t} = \tau\varphi + t, \tag{13}$$

where φ and t switch places. The term $\tau\varphi$ is very small because of the small value of τ.

In accordance with equation (13), consider the time and angle intervals. The time interval measured at the same value of angle ($\Delta\varphi = 0$) is completely determined $\Delta\tilde{t} = \Delta t$ whereas at the same time ($\Delta t = 0$) a time leap $\Delta\tilde{t} = \tau\Delta\varphi$ exists. The leap is the time of the rotation through angle φ at frequency $1/\tau$. Since equation (13) is the frame transformation into itself, the result may be interpreted as an uncertainty of the time determination. The maximal value of the leap is $2\pi\tau$ as at $\varphi = 2\pi$ the frame coincides with itself.

For further conclusions about the transformation, the function F must be defined by means of a physical principle.

If the second type truly corresponds to physical reality, then a lower limit for measurements of the characteristic constant may be drastically decreased.

3. Frequency superposition

Consider a plane circularly polarized light wave moving through an electro-optical crystal with the rotating optical indicatrix. The light and the indicatrix, for definiteness, are assumed to rotate in the same direction with frequencies ω and ν. In correspondence with equation (1), the optical frequency in the frame with the resting optical indicatrix is

$$\tilde{\omega} = \frac{\omega - \nu}{\sigma\Theta\omega/\tilde{\nu} - \nu/\tilde{\nu}}, \tag{14}$$

where $\omega = \varphi/t, \tilde{\omega} = \tilde{t}/\tilde{\varphi}$. Equation (14) is symmetric under the reverse transition, i.e. the change $(\tilde{\omega}, \tilde{\nu}) \longleftrightarrow (\omega, \nu)$ and, therefore, the transition produces the initial frequency ω at crystal output. However if the reverse rotation occurs, then we must make the change

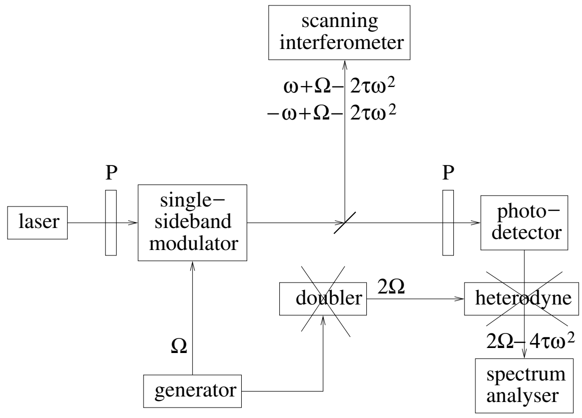

Figure 1. A schematic of the experiment. P is the polarizer.

$(\omega, \nu) \to (-\tilde{\omega}, \tilde{\nu})$ in the right part of equation (14). As a result we obtain, after simplification, the output frequency

$$\omega_{\text{out}} = \frac{-\omega + 2\nu - \sigma\Theta\omega}{-2\sigma\Theta\omega/\nu + \sigma\Theta + 1}. \tag{15}$$

It is obvious that for limiting values of ν_c, defined by the condition $\sigma\Theta(\tilde{\nu}_c\nu_c) = 1$, $\omega_{\text{out}} = \nu_c$ and $\tilde{\omega} = \tilde{\nu}_c$.

For the solution of the first type, $1 - 1/\tilde{q}q = \sigma\tilde{\nu}\nu$. Taking into account that $\nu \ll \omega$ and $\tilde{\nu} \approx -\nu$ for small ν, we obtain from equation (15) in the first approximation

$$\omega_{\text{out}} \approx -\omega + 2\nu \mp 2\tau^2\nu\omega^2. \tag{16}$$

The extra frequency shift $2\tau^2\nu\omega^2$ is equivalent to that in [7]. The shift cannot be measured optically because of the very small characteristic constant τ. Even if the modulating frequency is a powerful optical wave $\nu \sim 10^{14}$ Hz, then at $\tau \sim 10^{-23}$ s the extra shift for $\omega \sim 5 \times 10^{14}$ is $|2\nu\tau^2\omega^2| \sim 10^{-2}$ Hz. Such a shift cannot be picked out in laser or photodetector noise.

Consider the second type of solutions with $1 - 1/\tilde{q}q \approx \tau\sqrt{-\tilde{\nu}\nu}$. In the first approximation,

$$\omega_{\text{out}} \approx -\omega + 2\nu - 2\tau\omega^2. \tag{17}$$

The extra shift $2\tau\omega^2$ does not depend on ν, i.e. such a shift must be produced by the usual half-wave plate. Note that from the viewpoint of the Maxwell equation, the frequency shift in the single-sideband modulator is a consequence of the phase difference between the two components of the electric field of light wave whereas from the viewpoint of photons it is something different.

The extra shift may be interpreted as an energy of the polarization reversal. The sign difference of the energy corresponds to the assumption on inequivalence of the right and left rotations. The shift of the second type may far exceed the shift of the first type. The relative value of the extra shift for $\tau \sim 10^{-23}$ s is $2\tau\omega \sim 10^{-8}$ in the visible range.

4. Measurement of the extra shift

A schematic of the experiment for measuring the extra shift of the second type is shown in figure 1.

Linearly polarized light from a laser passes through the single-sideband modulator (for example, lithium niobate modulator [6]; the author of the given paper would like to hope that the method for measurements of such small optical shifts can stimulate technological efforts and bring into existence a waveguide single-sideband modulator with a low half-wave voltage and variety of applications). As noted in section 3, the half-wave plate may be used instead of the modulator; however, manipulations of the modulating frequency are required to enhance accuracy. An electric field rotating at frequency $\Omega = 2\nu$ is applied to the modulator. Evolution of the laser spectrum under a change of the amplitude and frequency of the electric field may be observed by means of the scanning interferometer. The linearly polarized light is a sum of two circularly polarized waves of frequencies ω and $-\omega$, respectively. The modulator changes the frequencies to $-\omega + \Omega - 2\tau\omega^2$ and $\omega + \Omega - 2\tau\omega^2$ respectively. The output from the polarizer is modulated in intensity at frequency $2\Omega - 4\tau\omega^2$. The extra shift $4\tau\omega^2$ could be extracted by heterodyning as in [7]. However in the case described here, the schematic is simplified (heterodyne and doubler are crossed out in figure 1) since the resonance $\Omega = 2\tau\omega^2$ can be used by matching the sign of τ by the reversal of the applied rotating electric field.

5. Conclusion

The idea of inequivalence of the direct and reverse frequencies (is parity violation connected with such an inequivalence?) and the symmetry principle leads to three types of transformation for point rotation frames. The first type is a generalization of the Lorentz transformation. This type seems more applicable to the Cartesian frames as it follows from some symmetry considerations. Measurements of the extra frequency shift in this case lie beyond the possibilities of optics. Allowable values of frequencies for the second and third types are bounded above and for the third type also below. Therefore, only the second type is applicable to the electromagnetic field. Different solutions in the third type of transformation may be associated with different fields and the lower frequency boundaries with the field masses. Unusual and strange properties of the second type are uncertainty of the time determination and the fact that the extra shift does not depend on the modulating frequency in the first approximation. The shift may be measured if the characteristic constant in the transformation is about 10^{-23} s. The proposed resonance method may be effectively used for precision measurements in laser spectroscopy.

A key to completion of the above construction is the explicit form of the function $F(x)$. The function must equal zero for $x = 0$, must have, as a minimum, one negative root and a sufficient number of solutions of the third type.

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